

Euler and the number "e"

Etienne Brauns May 16, 2012

In Dr. Cliff Pickover's very interesting book "**The Math Book**" (see Cliff's website <http://www.pickover.com/>), I encountered again after many years, when reading the pages on Leonhard Euler, the astonishing equation combining e, i, π , 1 and 0 :

$$e^{i\pi} + 1 = 0$$

- e : [http://en.wikipedia.org/wiki/E_\(mathematical_constant\)](http://en.wikipedia.org/wiki/E_(mathematical_constant))
- i : http://en.wikipedia.org/wiki/Imaginary_unit#Definition
- π : <http://en.wikipedia.org/wiki/Pi>

Dr. Cliff Pickover points in his "**The Math Book**" to :

- David Darling who considers the number "e" to be probably the most important number in mathematics.
- Benjamin Pierce who stated that we cannot understand this equation, that we do not know what the equation's meaning is, but that we however proved the equation and therefore that we know that the equation must be true.
- Kasner and Newman who remarked that we can only show the equation but not go into the equation's implication and that the equation has a profound impact on the scientist and the mathematician

That astonishing equation was also presented to me, during my first university year as a civil engineer, by our professor of mathematics, prof. H. Florin. That was about 40 years ago but during those years I regularly mentioned that equation to other people as a result of the very peculiar simultaneous presence of **e, i, π , 1 and 0** in one very short equation. I reconstructed a few days ago the analysis that we learned from prof. Florin at that time, as you will see below, **but certainly have also a look at the end of this text ...** So here we go,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \dots \quad (1)$$

Therefore equation (1) can be used in the case of "i.x" as :

$$e^{ix} = 1 + \frac{ix}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!} + \dots \quad (2)$$

Thus, when implementing $i^2=-1$, $i^3=-i$, $i^4=+1$, $i^5=+i$, ... equation (2) develops into :

$$e^{ix} = 1 + \frac{ix}{1!} - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!} + i\frac{x^5}{5!} - \frac{x^6}{6!} - i\frac{x^7}{7!} + \dots \quad (3)$$

When rearranging equation (3) while grouping the terms with "i" and without "i" one obtains :

$$e^{ix} = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right) + i \cdot \left(\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right) \quad (4)$$

While having :

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad (5)$$

and also :

$$\sin(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad (6)$$

it is possible to implement equations (5) and (6) into equation (4) and therefore one obtains :

$$e^{ix} = \cos(x) + i \cdot \sin(x) \quad (7)$$

In the case of $x = \pi$:

$$e^{i\pi} = \cos(\pi) + i \cdot \sin(\pi) = -1 + i \cdot 0 = -1 \quad (8)$$

Thus :

$$e^{i\pi} + 1 = 0 \quad (9) \quad !$$

I would now like to stretch things a little bit further since one can use equation (9) and write :

$$e^{i\pi} = -1 \quad (10)$$

Consider taking the logarithm at the left of equation (10) :

$$\text{LnOfLeftPart} = \ln(e^{i\pi}) = i \cdot \pi = \sqrt{-1} \cdot \pi \quad (11)$$

Consider taking the logarithm at the right of equation (10) :

$$\text{LnOfRightPart} = \ln(-1) \quad (12)$$

Since $\text{LnOfLeftPart} = \text{LnOfRightPart}$ one can combine equations (11) and (12) to

$$\sqrt{-1} \cdot \pi = \ln(-1) \quad (13)$$

Thus :

$$\pi = \frac{\ln(-1)}{\sqrt{-1}} \quad (14) \quad !$$

Equation (14) is then definitely as "mystical" as equation (9) since the existing number π can be obtained by dividing two non-existing numbers... ! A paradigm that is questionable or not ? That is up to you. But maybe you could also have a look at the rest of the website www.absolute-relativity.be where several existing paradigms in the light phenomena physics are exposed and questioned by experimental evidence and by theory ... The flawed mix-up, now already existing for

hundreds of years, of the artificial mathematical "space" (x,y,z) (only existing in our minds) with "real" space (in which our planet, the moon, space ships, satellites, galaxies, photons, etc. ... are moving in) in the human mind is the crucial element causing flawed contemporary paradigms.

The mathematical challenge to correct those flaws is very large since reference frames (x,y,z) indeed have a real velocity in real space. That real velocity can be determined by using photons as described on the website www.absolute-relativity.be and thus a new mathematical reference frame transformation system (based on the real velocity measurement) should be introduced to counter the existing incorrect light paradigm based approaches. Those approaches are sufficiently accurate in many applications but it is indicated at the website that they are also totally inaccurate in specific applications. Even more important however are the theoretical implications exposing specific theories in physics to be wrong : see e.g. the critique on the paradigm based on the Michelson and Morley experiment or the critique on the light clock paradigm and other critiques at www.absolute-relativity.be.