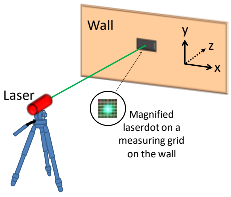

The Mercury perihelion precession: a critique on the anomaly and a plausible additional effect of the sun.

Private communication/publication

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1.0

Keywords: Mercury, perihelion, precession, anomaly, Newton, gravity, planetary system, sun, wobbling, Einstein, relativity, paradigm, light, photon, real space, real velocity, real location

Abbreviations: CS (contemporary science), CPBD (contemporary paradigms believer and defender), RS (real space), RV (real velocity), VS (virtual space), MWF# (My Website Figure) (a Figure at www.absolute-relativity.be; including references to dynamic Figures through their internet web link since it is not possible to directly implement dynamic/animated time stamp type of Figures in a Word or PDF based publication/document)

Dynamic figures in this publication and in the publications (1,2,3,4,5,6) are referred to as e.g. MWF2 (see *Abbreviations*). By clicking the link in Table 1 those dynamic figures will automatically open in your web browser.

Table 1 : dynamic MWF figure and the link

MWF#	Link
MWF2	www.absolute-relativity.be/images2/G6_Animation.gif

Note : A detailed discussion can be found within the extended publication (1) of over 400 pages which is downloadable at the website indicated in [a)]. The extended publication is informing in more detail about the existence/proofs of multiple flawed paradigms based on light/photons within CS and about important applications (on our planet and in space) resulting from those views. This publication and the preceding publications (2,3,4,5,6) are extracted from (1) and are intended as a series in the project indicated at ResearchGate entitled "*Karl Popper's type of falsification, through theoretical and experimental anomalies, of multiple contemporary paradigms based on light phenomena*". The information in (1),(2),(3),(4),(5),(6) and the website was registered in front of a notary and, in combination with the patent text, thus ensuring an author's copy right protection. The principle and result of the laser experiment (MWF2) was already published in a (notary registered) patent text and also already published at www.absolute-relativity.be.

a) *Private research contact* : all contacts should go through the Contact facility at the Home page of www.absolute-relativity.be

(1) Etienne Brauns, *A shattered Equivalence Principle in Physics and a future History of multiple Paradigm Big Bangs in "exact" science ?* ; this extended (notary registered) publication can be downloaded at http://www.absolute-relativity.be

(2) Etienne Brauns, *On multiple anomalies and inconsistencies regarding the description of light phenomena in contemporary science*
Website : http://www.absolute-relativity.be/pdf/MultipleAnomalies_EBrauns.pdf (version including the Annex)
Researchgate :
https://www.researchgate.net/publication/312190993_On_multiple_anomalies_and_inconsistencies_regarding_the_description_of_light_phenomena_in_contemporary_science
https://www.researchgate.net/publication/312591154_Annex_1_to_On_multiple_anomalies_and_inconsistencies_regarding_the_description_of_light_phenomena_in_contemporary_science

(3) Etienne Brauns, *On a massive anomaly through a straightforward laser experiment falsifying the equivalence principle for light.*
Website : http://www.absolute-relativity.be/pdf/ExperAnomLaser_EBrauns.pdf
Researchgate :
https://www.researchgate.net/publication/313030370_On_a_massive_anomaly_through_a_straightforward_laser_experiment_falsifying_the_equivalence_principle_for_light

(4) Etienne Brauns, *On the flawed Michelson and Morley experiment null-result paradigm*
Website : http://www.absolute-relativity.be/pdf/MichelsonMorley_EBrauns.pdf
Researchgate :
https://www.researchgate.net/publication/318969438_On_the_flawed_Michelson_and_Morley_experiment_null-result_paradigm

(5) Etienne Brauns, *On a flawed Lorentz contraction paradigm caused by an erroneous Michelson-Morley model and null-result.*
Website: http://www.absolute-relativity.be/pdf/Lorentz_EBrauns.pdf
Researchgate :
https://www.researchgate.net/publication/319128677_On_a_flawed_Lorentz_contraction_paradigm_caused_by_an_erroneous_Michelson-Morley_model_and_null-result

(6) Etienne Brauns, *On the inconclusiveness of the results from the Eddington 1919 solar eclipse mission to measure the bending of light.*
Website:
Researchgate :
https://www.researchgate.net/publication/319262673_On_the_inconclusiveness_of_the_results_from_the_Eddington_1919_solar_eclipse_mission_to_measure_the_bending_of_light

1. Abstract

More than a century ago the theoretical/calculated precession value (based on Newton's gravity laws) was considered to be "anomalous" with respect to the observed value of the perihelion precession of Mercury. There is a very small difference of only 43 arc seconds per 100 years between the observed/measured perihelion precession value (per 100 years only a minute 575 arc seconds = 0.159°, thus per year only 0.00159°) and the theoretically calculated value (per 100 years 532 arc seconds=0.147°; thus per year only 0.00147°). Einstein suggested in 1915 that this "anomaly" was also a test case for his relativity theory. Einstein derived later an equation from his relativity theory, predicting exactly the value of the "missing" 43 arc seconds, thus in an incredible precise way. In this publication however a critique on the contemporary Mercury perihelion precession "anomaly" paradigm is given. At

first it should be stressed that in contemporary science there is simply no exact solution (model) for Newton's gravitational configuration of a N-body system (N>2; thus also not for the system of the 8 planets and the sun). A model calculation evidently should be able to save the real phenomena occurring in real space. Therefore the question can definitely be raised what the meaning is of the "calculated" value of 532 arc seconds per 100 years (being calculated at that time) regarding its degree of "saving the real precession phenomenon in real space of the perihelion of Mercury". If the latter calculated value in fact thus originates from a theoretical/mathematical non-exact solution (model calculation producing a non-exact model value) and if there isn't any reference value from an exact solution: how can one consider an "anomaly" to exist between the observed value and a "calculated value"? Secondly and even more important it seems that the calculated value was generated (moreover at that time since Einstein already used that value) by model calculations in which the sun was considered to be that massive that the assumption was made that the sun can be considered to be in a fixed position in the model. In the literature therefore even only an oblateness of the sun is mentioned as being studied in the past in eventually trying to explain the Mercury "anomaly". It thus seems that contemporary science did not consider much more than an oblateness of the sun with respect to a possible additional role of the sun in the perihelion precession of Mercury. As a result, a study was undertaken to investigate the validity of the simple CS assumption of a fixed position of our sun in the configuration in real space of the 8 planets and the sun. An Euler based method was therefore applied to model the orbits of the planets in the solar planetary system without the assumption of a fixed sun. These model/simulation results reveal that our sun cannot be considered as to be located in a rigid position, as assumed wrongly in the (contemporary) calculations with respect to the precession of the Mercury perihelion. A contemporary approach in placing the sun in a rigid position in the origin of the model reference frame is then flawed. When implementing in the Newton's gravity based mathematical model the sun to participate in the global movement of the celestial bodies an eventual significant wobbling effect of the sun in real space shows up. The expected wobbling effect of the sun is larger than the sun's diameter according to the preliminary model results (thus of course much more important than a marginal oblateness of the sun). It is thus suggested that specialized centre's should re-calculate in much more detail in a three-dimensional model on more powerful computers the expected significant wobbling effect and to investigate the possibility of a sun's wobbling as a plausible real cause of the so-called perihelion precession "anomaly" of Mercury.

2. A critique on the contemporary Mercury perihelion anomaly paradigm and a suggestion with respect to another possible and plausible cause.

In ((1); section 12.9) one can find the detailed discussion about the Mercury perihelion anomaly. Therefore, in this publication a somewhat shortened version of that section 12.9 is given in Appendix I. However the same indexing as used in section 12.9 in (1) is retained in Appendix I, therefore also the same indexing for all equations. The reader is thus fully referred to Appendix I which reveals an expected severe wobbling in RS of the sun as the result of an obvious global pulling force exerted on the sun by the 8 orbiting planets; a global resulting pulling force which fluctuates with the changing configuration in time of the planet locations in RS. According to the preliminary model results that wobbling effect of the sun is larger than the sun's diameter! Such wobbling behaviour of the sun could of course have a much larger effect on the perihelion precession of Mercury than an extremely marginal oblateness of the sun. It is thus suggested that specialized centre's should investigate in much more detail in a three-dimensional model on more powerful computers that significant wobbling effect and thus to investigate the possibility of a sun's wobbling as a plausible real cause of the so-called perihelion precession anomaly of Mercury.

3. Conclusions

It is a fact that an exact solution of a gravitational N-body system (N>2) simply does not exist. A 3-body system shows already chaotic characteristics. The, at that time, calculated value of the perihelion precession of Mercury of only 532 arc seconds per 100 years is thus surely not based on an exact Newton's laws based solution of the 9-body configuration of the 8 planets and the sun since such exact model simply is unavailable. In fact, our planetary system could be considered to be "rather stable" but nevertheless still shows chaotic characteristics. From the mere lacking of a Newton's laws based exact model, the question is thus raised what the value of 532 arc seconds per 100 years in fact represents with respect to the real precession of the perihelion of Mercury in real space. To which extent does that value saves the real phenomenon of Mercury's precession in real space? From that uncertainty the relevance of the "anomaly" between the observed value and that "calculated" value in principle thus already can be questioned. Moreover, it seems that in obtaining that model value of 532 arc seconds per 100 years the simple assumption by contemporary science that the sun is in a fixed position is flawed. A straightforward simulation based on an Euler method reveals a possible wobbling effect of the sun being larger that the sun's diameter! It seems that such important effect was never considered or implemented in any of the calculations (at that time) producing the Mercury precession value of 532 arc seconds per 100 years. Therefore such stresses the question about the relevance of the latter value of 532 arc seconds and thus should thus initiate a re-investigation.

Appendix I

12.9 Critique on the Mercury perihelion "anomaly" and a possible/plausible additional effect of the sun.

12.9.1 Introduction

Our planetary system consists of 8 planets: Mercury (closest to the sun), Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. The 9th outer "planet" Pluto has a dwarf planet status and is not considered further here. Some main (orbit) data are presented in Table 12.1.

With respect to Mercury, two perihelion precession values exist in contemporary science:

a) the observed/measured value

b) the calculated one on the basis of Newtonian mechanics

Both values are e.g. reported at:
https://en.wikipedia.org/wiki/Tests_of_general_relativity#Perihelion_precession_of_Mercury

On one hand the perihelion precession of Mercury was thus claimed to be measured as **575 arc seconds per century**. The perihelion precession is thus an extremely tiny shift of only 0.1597° per 100 years, thus only 0.001597° per year (only 5.75 arc seconds per year)!

On the other hand one has thus the calculated perihelion precession value based on the Newtonian mechanics theory (F=m.a describing the acceleration of a body of mass m when applying a force F and also the inter-attraction force between two bodies of masses m_1 and m_2 according to $F=G.m_1.m_2/r^2$ with G the universal gravity constant and r the distance between the two bodies). However, in 12.9.2 the well known impossibility to exactly solve mathematically the N-body Newtonian mechanics problem of having a sun with 8 planets orbiting that sun is indicated! Notwithstanding the fact of the nonexistence of an exact mathematical solution for the N-body problem it is nevertheless claimed in literature that Newtonian mechanics "*taking into account all the effects from the other planets*" predict a precession of only **532 arc seconds per century**. It should be noted that the type of calculations that e.g. Le Verrier (http://adsabs.harvard.edu/abs/1859AnPar...5....1L) reported in 1859 regarding Mercury were performed on the basis of manual calculations.

There is thus a discrepancy of about 43 arc seconds per 100 years between the observed value of 574 arc seconds per 100 years and the calculated value of 532 arc seconds per 100 years. This discrepancy was called an anomaly since it was stated that Newtonian mechanics only resulted in 532 arc seconds per 100 years and thus could not predict accurately the measured value of 575 arc seconds per 100 years. It was Einstein who then came up with a relativity based calculation that perfectly matched the missing 43 arc seconds. Please note that the data "*observed value of 574 arc seconds per 100 years*" and "*calculated value of 532 arc seconds* *per 100 years*" are there already for over a century. The value of "*532 arc seconds per 100 years"* thus must have been "calculated" more than a century ago: Einstein indeed tackled that "*anomaly of 43 arc seconds*" as a relativity case and worked for many years (it seems it took him three years and it seems that it nearly exhausted him) before he obtained the equation which exactly results in the "missing" 43 arc seconds per 100 years... Perceive here that a full orbit is 360° which is 1 296 000 arc seconds. The claimed deduced difference and "anomaly" is thus only 0.43 arc seconds per year which is accordingly only 0.00003 **%** of a complete orbit. Both numbers are dazzling small! Einstein's relativity calculation thus arrived precisely/perfectly at that dazzling small amount of a "missing" 43 arc seconds per century between the theoretical modeling result and the observed value. One should reflect on the incredible accuracy in this case of such a minuscule "anomaly" of 0.43 arc seconds per year between such a minuscule observed value (5.74 arc second per year) and such a minuscule calculated value (5.32 arc seconds per year). That incredible result was and still is considered as an additional proof for Einstein's relativity theory.

Table 12.1

	Orbit duration	Orbits per year	Mass
	(days, approx)	(#)	(kg)
Sun			1.98910E+30
Mercury	88	4.15	3.30220E+23
Venus	224	1.63	4.86760E+24
Earth	365	1.0000	5.97219E+24
Mars	687	0.53	6.41850E+23
Jupiter	4332	0.084	1.89860E+27
Saturn	10832.33	0.0337	5.68460E+26
Uranus	30799.09	0.0119	8.68100E+25
Neptune	60190	0.0061	1.02430E+26

	Perihelion	Velocity @ Perihelion	Aphelion	Velocity @ Aphelion
	(m)	(m/s)	(m)	(m/s)
Sun				
Mercury	4.60012000E+10	58980	6.981690000E+10	38860
Venus	1.07477000E+11	35260	1.089390000E+11	34790
Earth	1.47098290E+11	30290	1.520982320E+11	29290
Mars	2.06669000E+11	26500	2.492093000E+11	21970
Jupiter	7.40573600E+11	13720	8.165208000E+11	12440
Saturn	1.35357296E+12	10180	1.513325783E+12	9090
Uranus	2.74893846E+12	7110	3.004419704E+12	6490
Neptune	4.45294083E+12	5500	4.553946490E+12	5370

12.9.2 The orbits of our planetary system

With respect to our sun and the orbits of its planets, from Newton’s analysis and gravitational

laws it was thought for a very long time that the system of planet orbits was running like a perfect clockwork. It was also thought at that time that such systems were totally deterministic, thus that it was able to calculate any orbit from Newton's laws and corresponding differential equations. Decades of mathematical analysis and research however revealed that such early over-optimistic point of view of being able to find the exact solutions for the N-body (see Wikipedia "N-body_problem") gravitational problem needed to be vastly attenuated. In fact only for a two body system an exact solution can be presented, a really meagre result! For the "*Gravitational three body problem*" Poincaré showed that the problem is not integrable; see also:
http://www.phys.lsu.edu/faculty/gonzalez/Teaching/Phys7221/ThreeBodyProblem.pdf.
Trying to solve a three body system is already vastly complex and the solution shows chaos characteristics (see https://en.wikipedia.org/wiki/N-body_problem#Three-body_problem and the animation there).

There is thus simply no exact/analytical solution for our planetary system! One should be well aware of the fact that our planetary system is thus not a meticulously running clockwork and that in reality it still shows chaotic process characteristics. Perhaps after that many billions of years of evolving into some kind of "stable" system but it should however be clear that the system is not in perfect balance. The orbits are not rock steady anchored in space in one univocal elliptical trajectory and still show a "degree of chaos" with respect to the still progressing orbit trajectories. The fully three-dimensional case of the configuration of the 8 planets with their individual orbit planes (at different angles from another) makes the calculations of the real orbits even much more complicated. In fact it is impossible to exactly calculate those 3D orbits. As ever, we humans try to grasp with mathematical means that reality into mathematical models. Mathematics is indeed a fabulous tool in trying to approximate reality (see e.g. my publication "*Finite elements-based 2D theoretical analysis of the effect of IEX membrane thickness and salt solution residence time on the ion transport within a salinity gradient power reverse electrodialysis half cell pair*" which you can download at:
- http://www.reapower.eu/publications.html
or at ResearchGate:
https://www.researchgate.net/publication/257414176_Finite_elements-based_2D_theoretical_analysis_of_the_effect_of_IEX_membrane_thickness_and_salt_solution_residence_time_on_the_ion_transport_within_a_salinity_gradient_power_reverse_electrodialysis_hal).

When however using such mathematical approach/modelling one must realize that the approach is always an approximation of reality, which may happen to be a very good approximation in a number of cases. However, anyone needs to be very prudent in claiming that calculation results exactly mimic reality. In that respect one should also be very careful not to claim neither the "accuracy" of mathematical model calculations results as to represent reality in an univocal link between the real phenomena in RS and the mathematical "variables" inside the mathematical modelling space.

In chapter 7 [*in (1)*] on the equivalence principle the basic gravity principles were explained regarding the gravity interaction of two material objects. A material object with index "i" and with mass M_i will attract another material object with index "j" with mass M_j and vice versa. In the system of the sun attracting all planets in our planetary system, while keeping them in orbit, there are numerous gravity interactions since planet i will also attract planet j. When including the sun there are thus 9 celestial bodies interacting with one another resulting in 8+7+6+5+4+3+2+1=36 interactions M_i<=>M_j. Each of these 9 celestial objects is thus subjected to the gravitational attraction of the 8 other celestial objects. Therefore each celestial object is subjected to 8 attraction forces. In total there are thus 9*8=72 gravitational forces acting. Evidently 72 gravitational forces correspond with the 36 interactions (M_i<=>M_j) x 2 forces (F_{ij}=F_{ji}).

These inter-planet gravitational actions need to be included in the calculation of the orbits. Needless to state again that an exact analysis or an exact analytical mathematical solution does not exist. Therefore, what is then the approach in science in trying to calculate the orbits ? That calculation approach is based on the numerical integration of the according differential equations on the basis of Newton’s laws and thus, by definition, only approximate the real orbits.

Moreover, regarding the modeling approaches one can read at the internet: "*Since the Sun is by far the most massive object in the Solar System, most discussions use a Copernican coordinate system with the Sun fixed at the origin*". Up to now it is thus assumed that our sun is that massive that it can be considered as being fixed at the origin of the mathematical/virtual "space" in which one is modelling mathematically the planetary system, thus also in the calculation of Mercury's perihelion precession. At the best and with respect to the latter, it thus seems that up to now one only considered the effect of an oblateness of the sun. But such oblateness was found to cause only a marginal effect on the precession of the perihelion of Mercury (0.0254 arc seconds per century as claimed in literature).

Consequently the stringent remark here: one was/is thus not considering at all a displacement in RS of the sun itself in the total configuration of 8 planets in the 9-body system (displacement as a result of the combined pulling forces of the 8 planets)? Therefore, in the next sessions a possible answer on that matter is given, based on a mathematical/numerical two-dimensional model of the gravitational movements of **9** masses. The result of that modelling exercise is shown to be very challenging as an eventual/plausible cause of the "*calculated value of 532 arc seconds per 100 years and considered to be anomalous* ".

12.9.3 Mathematical approach to approximate the gravitational induced celestial behaviour of a 9-body system

As already indicated, the basic equations which are used in the calculation of the orbits of celestial objects are Newton's equations:

$F = m \cdot a$ (12.9.1)

and:

$$F_{i,j} = F_{j,i} = G \cdot \frac{m_i \cdot m_j}{r^2} \quad (12.9.2)$$

with:
F = force exerted (N)
a = acceleration (m/s²)
m = mass of the object (kg)
and:
$F_{i,j} = F_{j,i}$ = gravitational force exerted between two objects with indices "i" and "j" ($F_{i,j}$ =the gravitational force exerted by object "j" on "object "i" and vice versa)
G = universal gravitation constant (6.67384 E-11 $m^3.kg^{-1}.s^{-2}$)
m_i and m_j = masses of respectively object "i" and "j"

It is common to use a polar coordinate system (r,θ) to model the orbits of a set of celestial bodies but I decided not to use the polar coordinate system but a classic Cartesian coordinate system. Moreover, while still enabling to make my point here, I will "simplify" the problem of modelling the movement of a set of 9 gravitational inter-acting objects/masses to a two-dimensional case of those masses, having those objects thus moving ("orbiting") in one plane (x,y). The main conclusions will be however the same for the even more complicated three-dimensional case where the celestial bodies have their own particular orbital plane in RS.

I will introduce here thus a model system of a massive object around which 8 other masses will orbit as a result of gravity. I will call the "central" object Sol having the same mass as our sun. The other objects are called Mer, Ven, Ear, Mar, Jup, Sat , Ura and Nep. At time t=0 these objects are respectively placed at the right of Sol at distances, with respect to Sol, according to the aphelion distances of the real planets Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune and in the same order.

Table 12.2

Index of Object	Name of Object	Mass	x-location @ t=0	y-location @ t=0	v_y @ t=0	v_x @ t=0
		(kg)	(m)		(m/s)	(m/s)
1	Sol	1.98910E+30	0	0		
2	Mer	3.30220E+23	6.981690000E+10	0	38860	0
3	Ven	4.86760E+24	1.089390000E+11	0	34790	0
4	Ear	5.97219E+24	1.520982320E+11	0	29290	0
5	Mar	6.41850E+23	2.492093000E+11	0	21970	0
6	Jup	1.89860E+27	8.165208000E+11	0	12440	0
7	Sat	5.68460E+26	1.513325783E+12	0	9090	0
8	Ura	8.68100E+25	3.004419704E+12	0	6490	0
9	Nep	1.02430E+26	4.553946490E+12	0	5370	0

It is assumed here and on an arbitrary basis that at t=0 the mass centre's of the objects Sol, Mer, Ven, Ear, Mar, Jup, Sat, Ura and Nep are perfectly aligned and positioned on the abscissa x. At t=0 the objects Mer, Ven, Ear, Mar, Jup, Sat, Ura and Nep are also given an initial and immediate velocity purely in the positive y-direction (see values in Table 12.2). The orbits of Mer, Ven, Ear, Mar, Jup, Sat, Ura and Nep thus start in their location indicated in Table 12.2 in their counter clock wise orbits around Sol at the indicated "launching" speeds. In physics it is common to use vector notations for phenomena such as force and velocity. Such vector can be decomposed in the x- and y-direction in a two-dimensional case. As an example a velocity vector **v** can be de decomposed into $\mathbf{v_x}$ and $\mathbf{v_y}$ showing scalar values v_x and v_y. From those v_x and v_y scalar values the vector **v** evidently can be re-assembled. In the Mer, Ven, Ear, Mar, Jup, Sat, Ura and Nep (including t=0) Table 12.2 the initial v_x and v_y scalar values at t=0 are indicated.

How can one now tackle the calculation of the orbits of those objects around Sol?

As a first indication of the way mathematicians tackle the solving of differential equations through numerical methods a very relevant and "simple" example here is that of a body showing a mass m being accelerated along an axis "x" at an acceleration rate "a" caused by a constant force F. So one has:

$$F = m \cdot a$$

In this case one has (with v = velocity):

$$a = \frac{d^2x}{dt^2} = \frac{d}{dt}(\frac{dx}{dt}) = \frac{d}{dt}(v) = \frac{dv}{dt} \quad (12.9.3)$$

One is interested here in trying to get a solution for v(t) as well as x(t).

From the point of view of velocity:

$$F = m \cdot a = m \cdot \frac{dv}{dt} \quad (12.9.4)$$

thus:

$$F \cdot dt = m \cdot dv \quad (12.9.5)$$

or:

$$dv = \frac{F}{m} \cdot dt \quad (12.9.6)$$

After integration (since F is constant in this example) :

$$v = \frac{F}{m} \cdot t + v_0 = a \cdot t + v_0 \quad (12.9.7)$$

Therefore, in the case of an object with a mass m and being subjected to a constant force F the velocity will increase linearly with time.

From the perspective of the location "s" of the object:

$$v = \frac{ds}{dt} = \frac{F}{m} \cdot t + v_0 = a \cdot t + v_0 \quad (12.9.8)$$

thus:

$$ds = \frac{F}{m} \cdot t \cdot dt + v_0 \cdot dt \quad (12.9.9)$$

After integration (F and m are constant):

$$s = \frac{F}{m} \cdot \frac{t^2}{2} + v_0 \cdot t + s_0 \quad (12.9.10)$$

or:

$$s = a \cdot \frac{t^2}{2} + v_0 \cdot t + s_0 \quad (12.9.11)$$

This is the well known equation describing the trajectory of an object with a mass m, being subjected to a constant force F (thus a constant acceleration a).

For such a simple case there is evidently an exact solution; exact equations (12.9.11) and (12.9.7) for trajectory s(t) and velocity v(t). There are however alternative mathematical ways to "solve" the differential equations related to velocity "v" and trajectory "x" or in general "to solve" differential equations which are more difficult or even impossible to obtain an analytical solution for. In that respect numerical approaches such as e.g. the Euler method or e.g. the Runge-Kutta (RK2, RK3, RK4) methods are an option (next to many other numerical methods to approximate the solution of a differential equation or a set of differential equations).

Before going into details of the Euler method or the Runge-Kutta methods, let's have first a look at the principles of an approximation through a Taylor's series (expansion) in the case of a trajectory function s(t):

$$s(t + \Delta t) = s(t) + s'(t) \cdot \Delta t + \frac{1}{2} \cdot s''(t) \cdot \Delta t^2 + \frac{1}{3!} \cdot s'''(t) \cdot \Delta t^3 + \ldots \quad (12.9.12)$$

We remark that in the case of the object with a mass m and subjected to a constant force F (thus constant value of acceleration a) that s'(t)=v(t) and s"(t)=a while higher order differentials such as s'"(t)=0 as a result of s"(t) being a constant, thus:

$$s(t + \Delta t) = s(t) + v(t) \cdot \Delta t + \frac{1}{2} \cdot a \cdot \Delta t^2 \quad (12.9.13)$$

When we apply the same reasoning to a velocity function v(t):

$$v(t + \Delta t) = v(t) + v'(t) \cdot \Delta t + \frac{1}{2} \cdot v''(t) \cdot \Delta t^2 + \frac{1}{3!} \cdot v'''(t) \cdot \Delta t^3 + \ldots \quad (12.9.14)$$

We remark again that in the case of the object with a mass m and subjected to a constant force that v'(t)=a and v"(t)=0 while higher order differentials such as v'"(t)=0 thus vanish, thus:

$$v(t + \Delta t) = v(t) + a \cdot \Delta t \quad (12.9.15)$$

Equations (12.9.13) and (12.9.15) are called numerical approaches since one calculates step-wise the next result. Starting from initial values one calculates for a step Δt the second status after Δt. Then a third status is calculated from the second status, again for a time step Δt and so on. When one compares the numerical approach expressed by equations (12.9.13) and

(12.9.15) with the analytical (exact solution equations) (12.9.8) and (12.9.11) it is clear that both will deliver the very same results in the case that an object of mass m is subjected to a constant force. Let's illustrate this with a simple example. Consider a falling object as a result of the gravitational action of the Earth (friction with the air is not considered here). The object falls from a height position of 5 m at time t=0 and at a starting velocity $v_{t=0}=0$. One first calculates the vertical fall trajectory s(t) as well as the velocity v(t) while using equations (12.9.8) and (12.9.11). The gravity acceleration a=9.81 m/s². In the analytical approach one simply implements the value of t in the equations (12.9.8) and (12.9.11) in order to directly obtain the values for v(t) and s(t). The results are shown in Table 12.3.

Table 12.3

		Analytical approach (equations (8) and (11))			Taylor series based approach (equations (13) and (15))	
t		v(t)	s(t)		v(t)	s(t)
0		0	0		**0 (initial)**	**0 (initial)**
0.05		0.4905	0.012263		0.4905	0.012263
0.1		0.981	0.04905		0.981	0.04905
0.15		1.4715	0.110363		1.4715	0.110363
0.2		1.962	0.1962		1.962	0.1962
0.25		2.4525	0.306563		2.4525	0.306563
0.3		2.943	0.44145		2.943	0.44145
0.35		3.4335	0.600863		3.4335	0.600863
0.4		3.924	0.7848		3.924	0.7848
0.45		4.4145	0.993263		4.4145	0.993263
0.5		4.905	1.22625		4.905	1.22625
0.55		5.3955	1.483763		5.3955	1.483763
0.6		5.886	1.7658		5.886	1.7658
0.65		6.3765	2.072363		6.3765	2.072363
0.7		6.867	2.40345		6.867	2.40345
0.75		7.3575	2.759063		7.3575	2.759063
0.8		7.848	3.1392		7.848	3.1392
0.85		8.3385	3.543863		8.3385	3.543863
0.9		8.829	3.97305		8.829	3.97305
0.95		9.3195	4.426763		9.3195	4.426763
1		9.81	4.905		9.81	4.905

When using the Taylor series approach however on the basis of a time step Δt=0.05 sec one needs to calculate first from the initial value v(0) the value of v(0.05) by using equation (12.9.15), thus by explicitly using the previous value v(0) and adding to v(0) the product of "a" and Δt. After obtaining in that way the value v(0.05) one can continue the numerical procedure by using v(0.05) as a "starting" value to calculate the next v(0.1) value by adding to v(0.05) the product of "a" and Δt and so on. It is clear from Table 12.3 that exactly the same values for v(t) are obtained in this case when applying the analytical equations or the Taylor series based approach.

Moreover, the same is true for the numerical calculation, thus on the basis of the Taylor series, of s(t) by using equation (12.9.13). Now however one needs in the first numerical calculation step both the initial v(0) and s(0) to implement in equation (12.9.13). Thus using the Taylor series approach on the basis of a time step Δt=0.05 sec one calculates first from the initial values s(0) and v(0) the value of s(0.05) by using equation (12.9.13), thus by:

- explicitly using the previous value s(0)
- adding to s(0) the product of v(0) and Δt
- adding to that sum the product $0.5*a*\Delta t^2$

After obtaining in that way the value s(0.05) one can continue the numerical procedure by using s(0.05) and v(0.05) as starting value to calculate the next s(0.1) value according to the same procedure. It is clear from Table 12.3 that exactly the same values for s(t) are obtained in this case when applying the analytical equations or the Taylor series based approach. The example shows that under conditions of a constant force (thus constant acceleration) the Taylor series based approach gives exactly the same result as the analytical based solution.

Knowing now such interesting fact: what about the Euler method to "solve" (/approximate) an ordinary differential equation (ODE)?

Consider a function u(t) which satisfies for all t in the interval $[t_0, t_{end}]$ the next ODE:

$$\frac{du}{dt} = f(t,u) \quad (12.9.16)$$

Moreover, there are initial values u(t0) and u'(t0) (if there would be no initial values there would be an infinite number of "solutions"). One can now apply a finite difference method (numerical method) such as the Euler method to approximate the correct solution u(t). If the ODE (12.9.16) would be analytically solvable one would obtain the correct solution u(t) and present it graphically as a reference to the approximative "solution" $u_a(t)$. In many cases there will be no such correct analytical solution (equation u(t)) and thus there is then no reference graphical representation of u(t). The finite difference method thus results in a function $u_a(t) \approx u(t)$.

In the case of the Euler method the finite difference method principle is straightforward.

$$u'(t) = \frac{du}{dt} = \lim_{\Delta t \to 0} \frac{\Delta u}{\Delta t} \quad (12.9.17)$$

$$u(t+dt) = u(t) + du = u(t) + \frac{du}{dt} \cdot dt = u(t) + u'(t) \cdot dt \quad (12.9.18)$$

$$u(t+\Delta t) \approx u(t) + \frac{\Delta u}{\Delta t} \cdot \Delta t \approx u(t) + u'(t) \cdot \Delta t \quad (12.9.19)$$

The initial conditions are given at t=0 : thus both the u(0) value and the u'(0) value are known. Thus we can start at t=0 with considering $u_a(0)=u(0)$ and $u'_a(0)=u'(0)$. It is then easy to obtain an approximated value $u_a(0+\Delta t)$ for $u(0+\Delta t)$ by calculating:

$$u(0+\Delta t) \approx u_a(0+\Delta t) = u_a(0) + u'_a(0) \cdot \Delta t \quad (12.9.20)$$

The equations (12.9.19) and (12.9.20) can be considered as Taylor series were we omit all terms from the second derivative onwards (thus at the right hand site of the square bracket within equation (12.9.21)):

$$u(t+\Delta t) = u(t) + u'(t)\cdot\Delta t + \left[\frac{1}{2}\cdot u''(t)\cdot\Delta t^2 + \frac{1}{3!}\cdot u'''(t)\cdot\Delta t^3 + ...\right] =$$
$$u(t) + u'(t)\cdot\Delta t + [\Theta] \qquad (12.9.21)$$

Obviously, the error Θ will get smaller the smaller one chooses the time step Δt. Interestingly, from this error Θ perspective the case of Newton's law F=m.a is a very particular ODE when having a situation where the force is constant, thus the acceleration is constant, thus the velocity is a linear function of time. Under such conditions the equation (12.9.15) for the velocity v(t) becomes exact. If we thus consider u(t)=v(t) while u'(t)=v'(t)=constant acceleration "a", then equation (12.9.19) can be read as:

$$u(t+\Delta t) = u(t) + u'(t)\cdot\Delta t = u(t) + a\cdot\Delta t \qquad (12.9.22)$$

thus since u(t)=v(t):

$$v(t+\Delta t) = v(t) + v'(t)\cdot\Delta t = v(t) + a\cdot\Delta t \qquad (12.9.23)$$

Obviously, equation (12.9.23) corresponds to equation (12.9.15). This remark is very significant here since further in this text the problem of the 8 celestial bodies orbiting a "central" massive body will be tackled. It is indeed very important to already realize at this stage that the change in velocity from v(t) to v(t+Δt) after a time interval Δt in the orbit of a celestial body orbiting the massive "central" celestial body can be calculated to a very high degree of precision in the case that Δt is selected to be sufficiently small in a way that the gravitational forces acting upon that celestial body during that small time interval Δt are very close to be constant. Under such conditions the acceleration "a" is also extremely near to be constant during that small time interval Δt. The Euler method applied for v(t) according to equation (12.9.23) is then a very good approximation when calculating the velocity of the celestial body. The Newtonian F=m.a particular equation as an ODE and the Euler method applied to "solve" the equation is thus a very good "match" when calculating (approximating very well) the velocity change during a small time interval Δt.

With respect to the calculation of the location shift from (x(t),y(t)) to (x(t+ Δt),y(t +Δt)) after the small time interval Δt, things are also very favourable with respect to the use of the Euler method. Before explaining such it is however necessary to picture somewhat more in detail what the orbiting of those 8 celestial bodies around the massive central body is all about. Specifically from the point of view of the two-dimensional example that will be looked into in this section.

Let's consider an object, in a two-dimensional case, subjected to the gravity forces of 8 other objects. The 8 forces exerted by those 8 objects are depicted in Figure 12.3A as force vectors F_i (i=1,...,8). These vectors F_i can be transformed into one resulting force vector F_r as depicted by the principle of vector summation within Figure 12.3B. As indicated in Figure 12.3C, the object is thus subjected to the resulting force F_r with respect to Newton's laws.

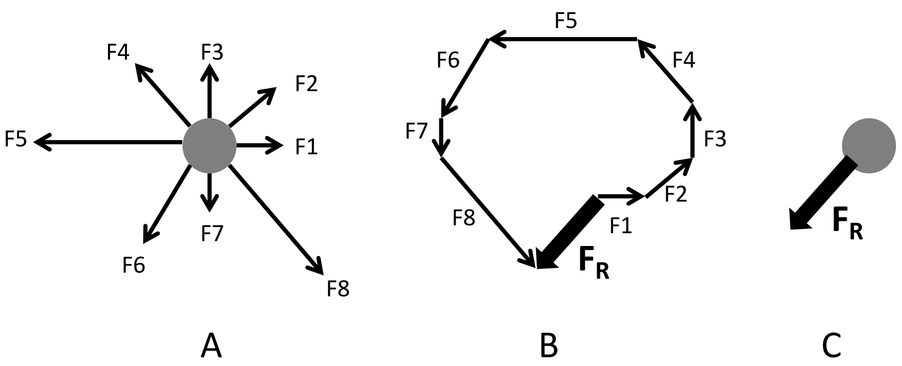

Figure 12.3 Force vectors and resulting force

Moreover, in a two dimensional case one can decompose the vector F_r in the x and y direction, as illustrated in Figure 12.4, into the vectors Fx_R and Fy_R. Decomposing the resulting force F_r thus allows to split the gravitational study into two cartesian directions x and y (instead of choosing for a polar coordinate system) and also to split the evaluation of the movement (acceleration, velocity, displacement) of the object in the (x,y) space into two separate parts : x-direction and y-direction. One thus can calculate in the Cartesian coordinate system from Newton's equations:

- Fx_R=m.a_x the values of the acceleration component a_x, the velocity component v_x and the displacement component x
- Fy_R=m.a_y the values of the acceleration component a_y, the velocity component v_y and the displacement component y

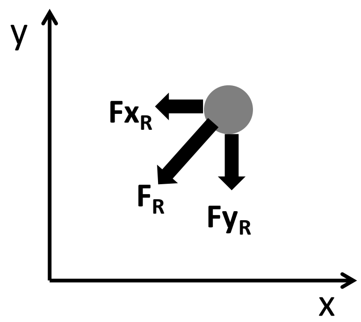

Figure 12.4 Decomposition in x- and y-component of F_R

It is thus possible to calculate (approximate) in this way the trajectory (x(t), y(t)) of the object as a result of the actual (Fx(t), Fy(t)) value on the basis of a finite difference method (such as the Euler method). It is very important to remark here that, when choosing a polar coordinate system, it is common to place the sun as a massive "central" object in the origin (0,0) and consider that massive "central" object as being fixed in (0,0). In such approach it is then only the movement (acceleration, velocity, displacement) of the objects, orbiting around the massive "central" object, one is dealing with. Accordingly, it is then also clear that one could overlook a possible important "taking part in the overall movements of the objects" of that so-called fixed mass which is moving itself also**.** When "expecting" the central object to be that massive that one can "neglect" its (the sun's) own movement under the action of the attraction forces by the orbiting planets and mistakenly consider the suns's movement as being nil: could that possibly induce an important Mercury perihelion modelling error? Is our sun indeed that massive that it can be really considered as being totally fixed in the mathematical model? Is it possible to introduce another modelling approach which, in contrast, points to the fact that our sun is indeed not fixed in space? That in reality our sun is in fact wobbling in space as a result of the gravity pulling forces, exerted by the planets on the sun? The effect of a wobbling sun can indeed be demonstrated by the two-dimensional modelling approach with the "celestial" bodies model Mer, Ven, Ear, Mar, Jup, Sat , Ura and Nep.

Since one deals with a configuration of 9 interacting objects there are 72 gravitational forces from 36 interactions with $F_{ij}=F_{ji}$ (i=1 to 9 and j=1 to 9 ; $i\neq j$). From any position configuration of the 9 objects in the (x,y) space one can then calculate through Newton's equation each of the 72 interaction forces $F_{ij}=F_{ji}$. After the calculation of all those forces it is then possible to calculate for each object "i" the actual resulting force $F_{r,i}$ acting upon object "i". After the calculation of all resulting forces $F_{r,i}$ (i= 1 to 9) acting upon each single object "i' it is possible to decompose each force vector $F_{r,i}$ into its two components $Fx_{r,i}$ and $Fy_{r,i}$. After obtaining those x and y direction force vector components it is then possible to calculate through Newton's basic law of the type of equation (12.9.1) the acceleration, velocity and displacement in the directions x and y. As a result of splitting the problem in the directions x and y, the relevant set of ODE's in a Cartesian type of two dimensional coordinate system (x,y) here are thus (9 objects thus for i= 1 to 9):

$$a_{x,i} = \frac{d^2x_i}{dt^2} = \frac{d}{dt}\left(\frac{dx_i}{dt}\right) = \frac{d}{dt}(v_{x,i}) = \frac{dv_{x,i}}{dt} = \frac{Fx_{r,i}}{m_i} \qquad (12.9.24)$$

$$a_{y,i} = \frac{d^2y_i}{dt^2} = \frac{d}{dt}\left(\frac{dy_i}{dt}\right) = \frac{d}{dt}(v_{y,i}) = \frac{dv_{y,i}}{dt} = \frac{Fy_{r,i}}{m_i} \qquad (12.9.25)$$

In order to calculate (approximate) the solution of this set of ODE's it is possible to use the Euler approach. Some, having a background in other difference approximative solution methods such as Runge Kutta (RK), will declare the Euler method to be less sophisticated or less accurate than RK. Such critique may be founded on a general basis for the total field of all possible ODE's. However, I have pointed above already to the fact that Newton's law (as also expressed in equations (12.9.24) and (12.9.25)) is a particular case in the field of ODE's. Also that, under the condition of a sufficiently small time interval Δt, the interaction forces $Fx_{r,i}$ and $Fy_{r,i}$ are "stable". One can thus state that, on the basis of equation (12.9.23), the velocity related equations (12.9.26) and (12.9.27) are excellent finite difference approximations.

For 9 objects, thus for i=1 to 9:

$$v_{x,i}(t+\Delta t) = v_{x,i}(t) + v'_{x,i}(t)\cdot\Delta t = v_{x,i}(t) + a_{x,i}\cdot\Delta t \qquad (12.9.26)$$

$$v_{y,i}(t+\Delta t) = v_{y,i}(t) + v'_{y,i}(t)\cdot\Delta t = v_{y,i}(t) + a_{y,i}\cdot\Delta t \qquad (12.9.27)$$

It is then also clear that the displacement related equations (12.9.28) and (12.9.29) of the type of the equation (12.9.13) are also excellent finite difference approximations (see substantial numerical proof later in this publication). Again, there are 9 objects thus for i=1 to 9:

$$x_i(t+\Delta t) = x_i(t) + v_{x,i}(t)\cdot\Delta t + \frac{1}{2}\cdot a_{x,i}\cdot\Delta t^2 \quad (12.9.28)$$

$$y_i(t+\Delta t) = y_i(t) + v_{y,i}(t)\cdot\Delta t + \frac{1}{2}\cdot a_{y,i}\cdot\Delta t^2 \quad (12.9.29)$$

Therefore Newton's law of the type (12.9.2) and the Euler finite difference method based equations (12.9.24), (12.9.25), (12.9.26), (12.9.27), (12.9.28) and (12.9.29) form the framework for calculating the movement related items (acceleration, velocity and displacement) of each of the 9 objects. It is to be remarked that each of those equations needs to be considered as 9-fold, as a result of the index "i". Also the implementation of the equation of type (12.9.2) is 9-fold, including the calculation of the resulting forces through an adequate addition of the force vectors. In combination with a small time step Δt it is clear that a massive amount of calculations needs to be performed within the Euler finite difference approach. Nevertheless, the implementation of those equations in a suitable software allows to approximate well the orbits of the objects Mer, Ven, Ear, Mar, Jup, Sat , Ura and Nep around Sol (Sol as a massive "central" attracting object).

With respect to the implementation of the equations (12.9.2) and (12.9.24) ... (12.9.29) approach: I decided to go for a double precision calculation within a Visual Basic code, including the Excel environment regarding data storage combined with the Excel graphical representations facilities, e.g. orbit graphics. Before giving some more details on my Euler method based calculations, further reference can also be made to the Runge Kutta methods as explained at e.g. Wikipedia ("Runge–Kutta methods"). In the case of Newton's F=m.a there is a set of two ODE's to consider. The first ODE handles the velocity function v(t) (equation (12.9.30)) while the second ODE handles the location in space (equation (12.9.31)):

$$\frac{dv}{dt} = \frac{F}{m} \qquad (12.9.30)$$

$$\frac{d^2x}{dt^2} = \frac{F}{m} \qquad (12.9.31)$$

When applying a Runge Kutta approach one must implement the method for both ODE's in parallel which can be done by e.g. a matrix calculation approach: e.g. the RK4 pdf article by C.J. Voesenek "*Implementing a Fourth Order Runge-Kutta Method for Orbit Simulation*" which can be downloaded: (http://spiff.rit.edu/richmond/nbody/OrbitRungeKutta4_fixed.pdf.)

I also implemented the RK method but that attempt seemed not to help much (more the reverse) with respect to better "solving" (approximating) the v(t) part of the problem. In my mind this has to do with the following. RK is also a finite difference method and which is claimed to be more accurate, in general, than the Euler approach. In my opinion however and as indicated above: when choosing a small Δt and when having the situation that the gravitational force acting upon an object is very stable during the small Δt, the Euler method and related equations as explained above are intrinsically linked to the v(t) modelling (see equations (12.9.14) and (12.9.15)). However, e.g. the RK4 method gives rise to the use of 4 slopes. As a result, the use of 4 slopes in the RK4 method could (in my opinion) eventually cause, in the particular case of the ODE represented by equation (12.9.30), specific fluctuations in trying to target the real solution value at $t+\Delta t$. In fact, one should be aware that RK4 is a smart mathematical procedure (no more, no less) in trying to get a better estimate of the value of the real solution value. In a way, the RK4's use of 4 slopes can be considered as a method which can produce an overestimated value as well as an underestimated value. One should thus also realize that there is some randomness involved with the RK4 procedure and not diverging away more is thus also not guaranteed. However, since I obtained more than reasonable results with the Euler method and since it delivered me a significant insight I did not dwell further on the RK methods. However, I welcome of course others to eventually implement such more sophisticated method in a 3-dimensional configuration in order to try to model our planetary system in the way that I did in a 2-dimensional approach for a fictitious configuration of Mer, Ven, Ear, Mar, Jup, Sat, Ura and Nep "orbiting" Sol.

Another important item in the modeling of a configuration of 9 "celestial" bodies is the conservation of energy. It is well known that in a 9-body problem the total sum of kinetic and potential energy needs to be constant. That principle is even known to be embedded within Newton gravitational laws and for the cases of an exact solution it would be very clear that this energy principle would be obeyed. However, as explained, the 9-body problem cannot be solved exactly. Nevertheless, the principle can be used advantageously during the progressive finite difference calculation progression as a control of numerical stability. As a result I also implemented additionally a total energy check in my calculations for each single Δt calculation step. I added up the potential and kinetic energy and made sure that the sum kept the same value within each single calculation step. In that way an eventual accumulating, numerically caused, run-away could also be controlled and halted. Notwithstanding the fact that the calculations were double precision based, the control of the total energy status was thus an additional numerical stabilizing factor during the millions of calculations which were needed over a total orbit time period of 100 years while using a small finite difference time step Δt (in my calculations I used Δt=300 seconds).

<u>12.9.4 Calculation results for Mer, Ven, Ear, Mar, Jup, Sat , Ura and Nep orbiting Sol and discussion</u>

My modelling effort with respect to the orbits of a set of 8 objects orbiting a massive "central" object (Sol) in fact targeted mainly an insight in the behaviour of that so-called massive "central" object Sol itself. From my calculation results I think that it will become clear that a definite re-consideration of the role of our sun in our real planetary system is needed with respect to Mercury. Up to now only an "oblateness" of the sun was looked into as a possible parameter in the Mercury perihelion precession anomaly and was found to have a marginal effect.

Intuitively any one is able to reflect on the vast role of the eight planets orbiting that central sun. The configuration of those planets indeed is continuously changing from the extreme configuration that those planets become "aligned" to a status of a random configuration around the sun. So, intuitively, one must recognize then also that our sun itself is subjected to (moreover fluctuating since the planets change position continuously) massive forces exerted by those planets. Since a force, when acting during a specific time on an object, induces an acceleration (and speed) according to Newton's law F=m.a it should be clear that one needs to look into the eventual summed effect of the forces exerted by the planets on the location of our sun. Up to now and with respect to the Mercury anomaly it seems that contemporary scientists were considering our sun that massive that our sun is in a "rigid"/"fixed" position. That must have resulted at that time (over a century ago) in a calculated Mercury perihelion precession value of 532 arc seconds per century, when based on Newton's mechanics.

However, my Euler method based model simulation results with respect to the model "planets" Mer, Ven, Ear, Mar, Jup, Sat, Ura, Nep and Sol clearly show that Sol is not present in a "fixed" location amid the orbiting planets. From that simulation result it can be concluded that there must be an effect on the sun's "central" location in RS and that such "central" location is not rigid. The effect is expected to be very significant since in the simulations with respect to Sol amid Mer, Ven, Ear, Mar, Jup, Sat, Ura and Nep the calculation result clearly shows a lateral movement of Sol's centre, being larger than Sol's diameter!

The simulation was thus based on the Euler approach as already explained in 12.9.3. It must be clear to you in the mean time that the sun's planetary system is not a "precision clockwork". In reality however it is a system which we may perceive as a rather stable ("in balance") system but rather needs to be thought of as some kind of a "balanced chaotic" system. After billions of years of the evolution of our planetary system to that actual status, which (deceivably) looks totally stable to the human mind, that system is still in (although very slow) progress since it was and still remains chaotic system in the end. Moreover, the sun's planetary system is in reality a three-dimensional system which is thus even more complex to calculate in an (x,y,z) simulation. I restricted the simulation of Mer, Ven, Ear, Mar, Jup, Sat, Ura, Nep and Sol to a two-dimensional system, thus as a (x,y) simulation.

One will immediately raise the next question: "*Which starting values were used in the simulation with respect to their location* (x_{start}, y_{start}) *in the 2D space and their velocity vector* (v_x,v_y) ? That question is indeed highly relevant and at the same time again reveals the chaotic nature of a planetary system! A planetary system is at any moment in time an evolving system and therefore millions of different starting/initial values could be selected with respect to a simulation. Each different set of starting values will evidently cause a different simulation result. Luckily we know our planetary system to be stable up to a specific level after those billions of years of evolution and therefore I used in my simulation the values as presented in Table 12.2. As indicated, I used a Visual Basic based code within Excel to perform the calculations. As explained, the code simulates (on the basis of the Euler approach) from the starting positions and the starting values the next positions (after Δt=300 sec) and next velocities within the configuration of the nine objects Mer, Ven, Ear, Mar, Jup, Sat, Ura, Nep and Sol. The procedure is then repeated by considering the calculated positions and velocities as new starting positions and velocities.

In order to have a first check of the performance of the code that I wrote in Visual Basic of course I plotted the orbit result obtained for the object Ear for the first year. Since the amount of calculated data is that immense to store in Excel evidently one has to optimize the Visual Basic code in order to limit the amount of calculated data which is going to be stored in Excel cells (to be used for e.g. orbit plotting purposes). I therefore programmed the code to sample/select and store about 100 sets of calculated results (time, location and velocity) within one orbit of each of the objects Mer, Ven, Ear, Mar, Jup, Sat, Ura and Nep. In the case of Ear that is of course 100 sets for 1 year. For Mercury, at four orbits per year, that is thus also 100 sets of data being stored but for a quarter of a year.

At first I thus would like to show the simulation result for the orbit of Ear in the very first year; please keep in mind the extremely complex and mutual force interactions over time of Mer, Ven, Ear, Mar, Jup, Sat, Ura, Nep and Sol calculation for Ear ("representing" our planet) during their orbits around Sol.

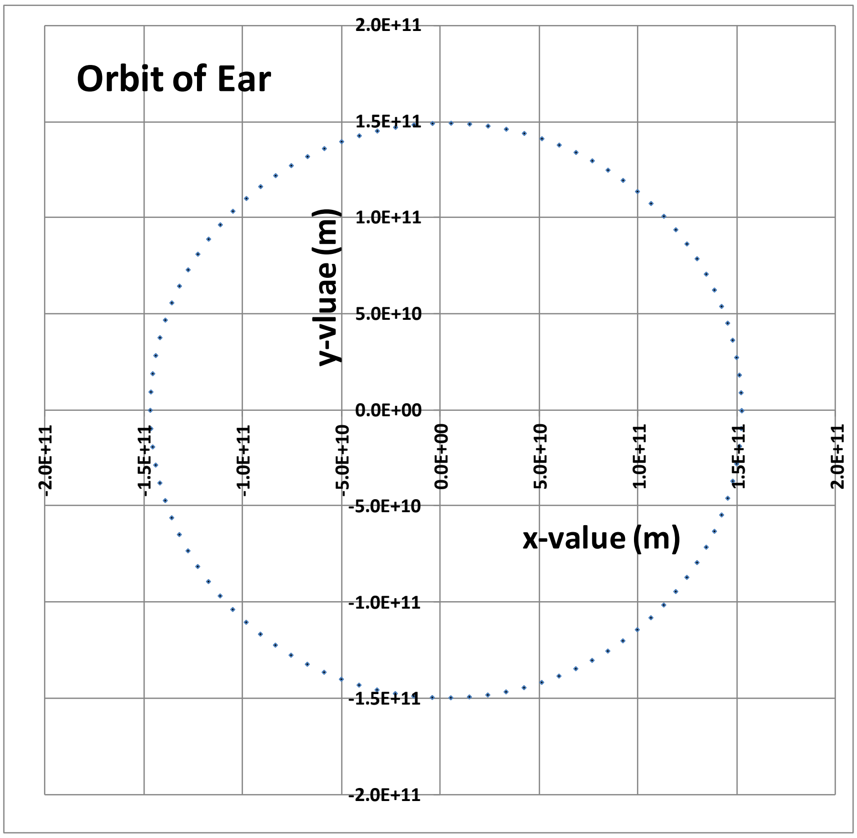

Figure 12.5 Orbit simulation of Ear

As the plot in Figure 12.5 shows, my simulation code worked very well since the Euler approach delivered immediately an Ear elliptic (near circle) orbit very close to our planet's orbit mentioned in literature, nonetheless the complexity of 9 interacting objects. That immediate result was already very supportive and encouraging. The result for Mer's first orbit was equally supportive, clearly resulting in coherent perihelion/aphelion simulation values close to the values mentioned in literature for Mercury. A simulation run was then executed with respect to a total period of a full 100 years (in one go; a few hours of calculation time by the computer). In that way a Mer, Ven, Ear, Mar, Jup, Sat, Ura, Nep and Sol simulation could be obtained revealing the total (theoretical) picture of orbits including an expected effect on Sol itself. It was indeed expected that Sol was not going "to sit still" in the origin (0,0) of the cartesian 2D-(x,y) frame and thus that Sol would participate in the total configuration of the interacting objects/masses.

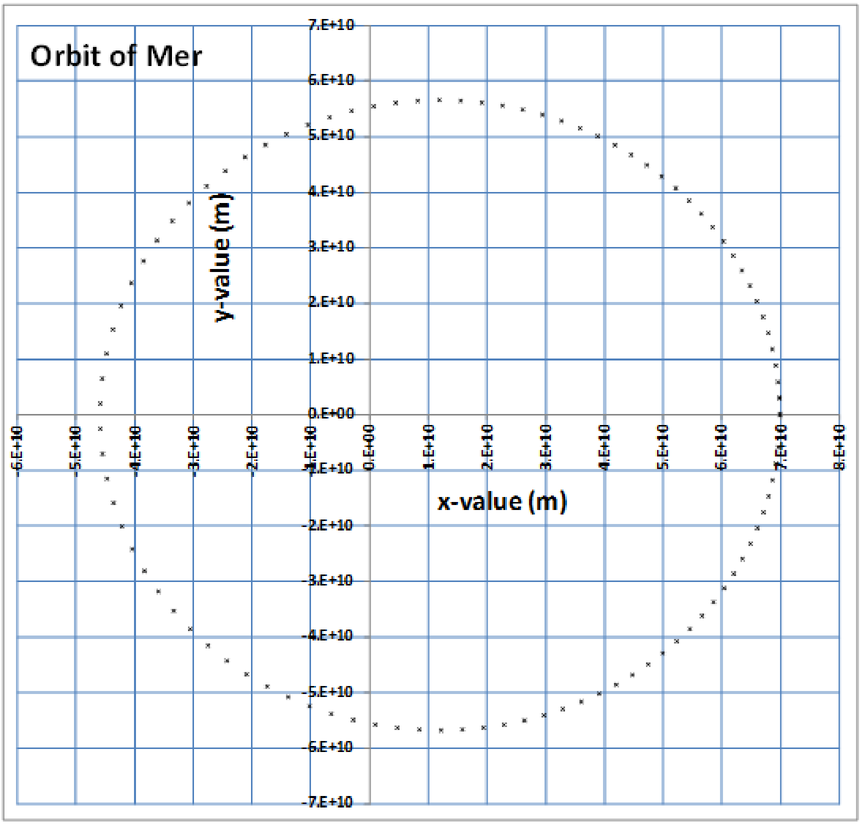

Figure 12.6 Orbit simulation of Mer

A third important element in this all is the total energy (total of potential and kinetic energy) of the configuration of 9 objects which should remain the same. Proof of that is demonstrated in Figure 12.7 Please note that potential energy of a mass in a configuration of multiple masses involves, by definition in physics, the work related in bringing the mass from an infinite distance toward its location in the configuration of other objects in space. Since the force, as a vector, being involved in implementing the object in the configuration, has an opposite direction as the displacement orientation the work (potential energy) is negative. Therefore the potential energy is indicated in Figure 12.7 as negative, conform to the conventions in physics. Please note that the total potential energy in absolute terms is about twice the value of the total kinetic energy and that the total energy (sum of potential and kinetic energy) is therefore "negative" in Figure 12.7.

It should also be clear that the total energy curve evidently is a flat line (since constant) but that such is not the case for the kinetic energy (KinEnergSolPlan in Figure 12.7) and for the potential energy (PotEnergTot). Indeed it is simple to understand that the status of an aligned configuration of the planets must be different from a well dispersed configuration of the planets around the sun.

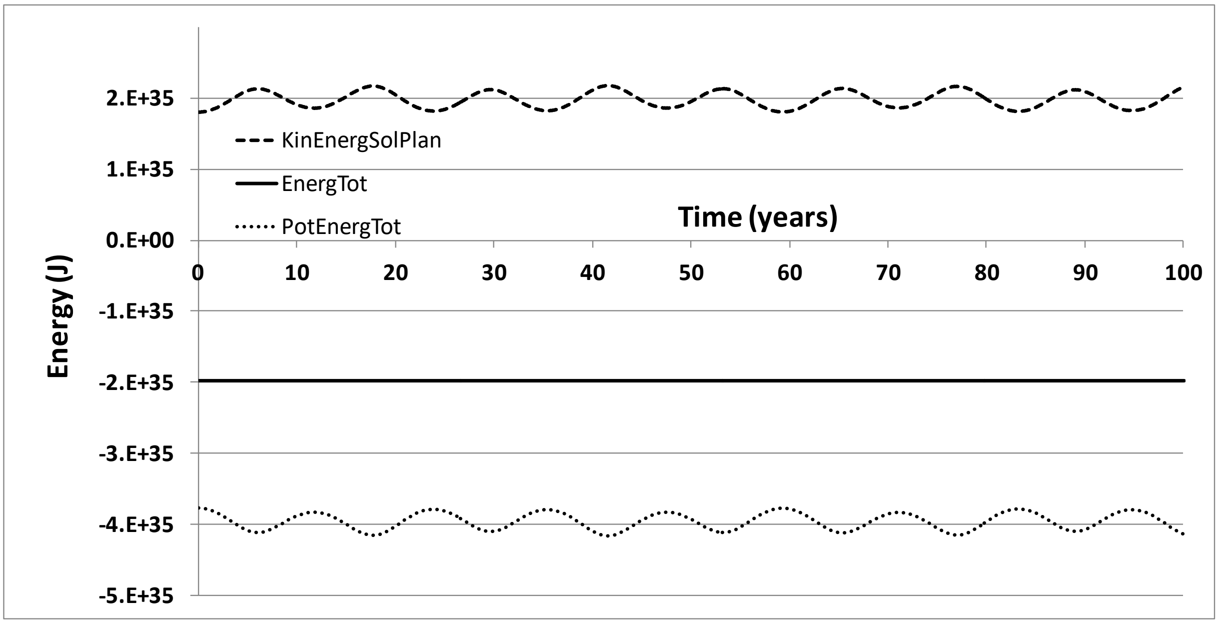

Figure 12.7 Kinetic and potential energy

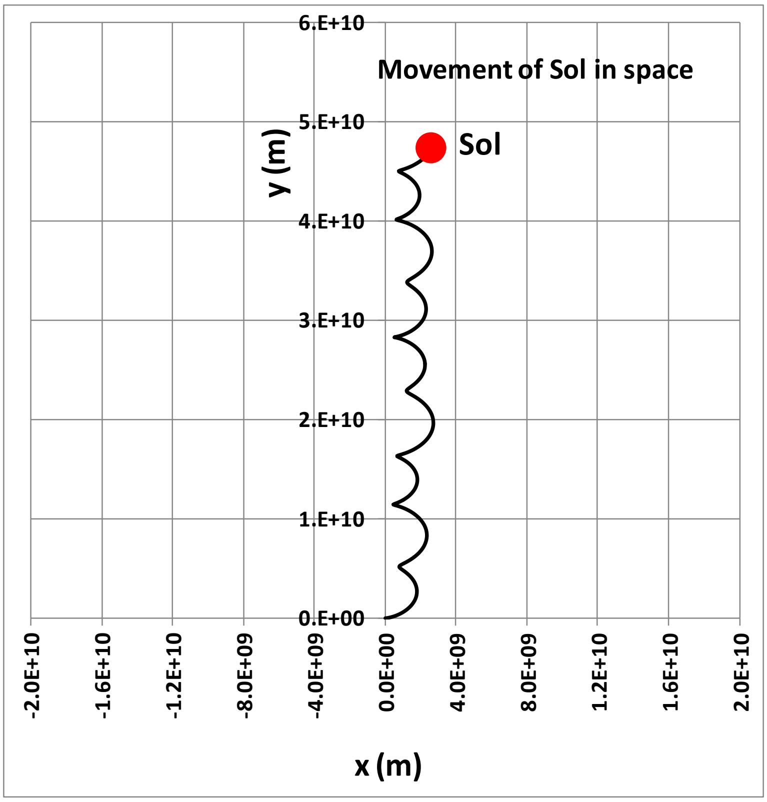

Figure 12.8 The simulation for Sol points to a very high probability of the sun's significant lateral displacement to be larger than its diameter

From an energy perspective some kind of oscillation thus is expected on beforehand. Figure

12.7 is clear enough in that respect. The simulation indeed shows the expected oscillations for the kinetic and potential energy (showing a perfect balancing of one another). The simulation results are therefore considered by me to be relevant with respect to our planetary system.

What then about the participation of Sol in the total configuration in RS? Is Sol "staying rigidly" in (0,0) all the time with negligible disturbances regarding that position. Is Sol that massive that it does not move at all?! The answer is given in Figure 12.8 where the diameter of Sol (in red) is depicted on a scale close to about its diameter (1.4E09 m). The 2D-simulation in Figure 12.8 thus clearly shows a 2D-movement of Sol over a period of 100 years. The result is thus that Sol is not rigidly fixed in (0,0) but definitely shows a significant fluctuating x-position of which the amplitude is larger than Sol's diameter!

Remark: one could wonder about the vertical movement of Sol in this simulation? In short: since I picked as starting conditions the ones in Table 12.2 there is also a vertical movement of Sol induced from these starting conditions (effect of Mer, Ven, Ear, Mar, Jup, Sat , Ura and Nep). That clearly needs to be eventually studied further in more elaborated simulations by others on more sophisticated computers which I do not own. It can be mentioned here that, in my simulation, Sol's vertical movement is parallel to a corresponding vertical and simultaneous movement by the planets themselves along with Sol. These multiple graphs are not integrated here in this publication since, in the end, the key point in Figure 12.8 is indeed the horizontal part of the displacement of Sol. Regarding Mercury's precession "anomaly" it is thus urgently needed to expand the analysis to this information regarding a plausible significant displacement of the sun itself. It could be very well the case that the "anomaly" of Mercury could be caused by a (to be expected on the basis of the Sol simulation results) wobbly behavior of our sun's location at the "centre" of the configuration of the nine celestial bodies. See also http://innumerableworlds.wordpress.com/2009/04/03/the-wobbling-sun/ pointing in the same direction. There one find also information for the wobbling of our sun and one can then estimate:

- indicated for a period of 50 years
- observed at a distance of 30 light years (that is 2.838E17m)
- from the graph: a wobble of about 0.0015 arc sec (that is 7.27E-09 radians)
- the value of the tangent of a very small angle (in radians) approximates very closely the value of the angle itself (in radians)
- thus in this case, the lateral wobble (m) displacement of the sun can be estimated* as distance (m) x wobble (radians) = 2.838E17m x 7.27E-09 = 2.0E09m

The estimated* wobble value of 2E09m supports my findings and therefore I suggest to the researchers to have a detailed look at all this information and to start a more profound simulation on powerful computers to check this all. Anyhow, the main problem here: one should keep in mind what is indicated in 12.9.1 about the experimental accuracy (disturbances) and theoretical accuracy (inexistence of an accurate model) in determining the very small yearly precession of Mercury's perihelion and the pitfalls in explaining the extremely small "difference" between, both very small, theoretical and experimental value.